Russell Kulsrud’s Contributions to Fusion Energy Research

Greg Hammett, May 16, 2026

A Day of Remembrance for Russell M Kulsrud (1928-2025)

https://web.astro.princeton.edu/day-remembrance-russell-m-kulsrud-1928-2025

4 Boys of Plasma Physics Theory (circa 1960s)

Martin Kruskal **Carl Oberman**

Russell Kulsrud **Norman Rostoker**

Russell memories

- My office was next door to Russell's for ~15-20 years.
- Constant stream of students and colleagues into his office, working through problems at the blackboard… He spent a lot of time working with students
- Asked lots of questions in the Theory Seminars. Never confrontational. Wanted to understand
- After he officially retired, kept working & contributing.
  At age ~75 (?) wrote textbook ("Plasma Physics for Astrophysics", 2004)
- Wasn't allowed to work on Fridays:
- Laney told him, "Russell, you're retired now! You can't work on Fridays anymore"
- Loved music. Piano lessons. Laney & he helped start the Princeton Festival (Opera every June)

Russell Kulsrud: 1993 recipient of the James Clerk Maxwell Prize for Plasma Physics

For his pioneering contributions to basic plasma theory, to the physics of magnetically confined plasmas, and to plasma astrophysics. His important work en-compasses plasma equilibria and stability, adiabatic invariance, ballooning modes, runaway electrons, colliding beams, spin-polarized plasmas, and cosmic-ray instabilities

(fusion topics in blue)

An energy principle for hydromagnetic stability problems

By I. B. Bernstein, E. A. Frieman, M. D. Kruskal and R. M. Kulsrud
Project Matterhorn, Princeton University

(*Communicated by S. Chandrasekhar, F.R.S.—Received* 18 *April* 1957—
Revised 26 *August* 1957)

The problem of the stability of static, highly conducting, fully ionized plasmas is investigated by means of an energy principle developed from one introduced by Lundquist. The derivation of the principle and the conditions under which it applies are given. The method is applied to find complete stability criteria for two types of equilibrium situations. The first concerns plasmas which are completely separated from the magnetic field by an interface. The second is the general axisymmetric system.

Proc.Royal Society 1958, https://doi.org/10.1098/rspa.1958.0023

THE PHYSICS OF FLUIDS VOLUME 1, NUMBER 4 JULY-AUGUST, 1958

Equilibrium of a Magnetically Confined Plasma in a Toroid*

M. D. Kruskal and R. M. Kulsrud
Project Matterhorn, Princeton University, Princeton, New Jersey
(Received May 27, 1958)

A variety of properties are derived satisfied by any static equilibrium of a plasma governed by the well-known magnetostatic equations. Some of these are local and quite trivial. Others involve integrals over surfaces of constant pressure, which are shown to be topologically toroidal under fairly general assumptions.

A variational principle for such equilibria is derived. One of its consequences is to provide a characterization of equilibria by their values of certain invariants.

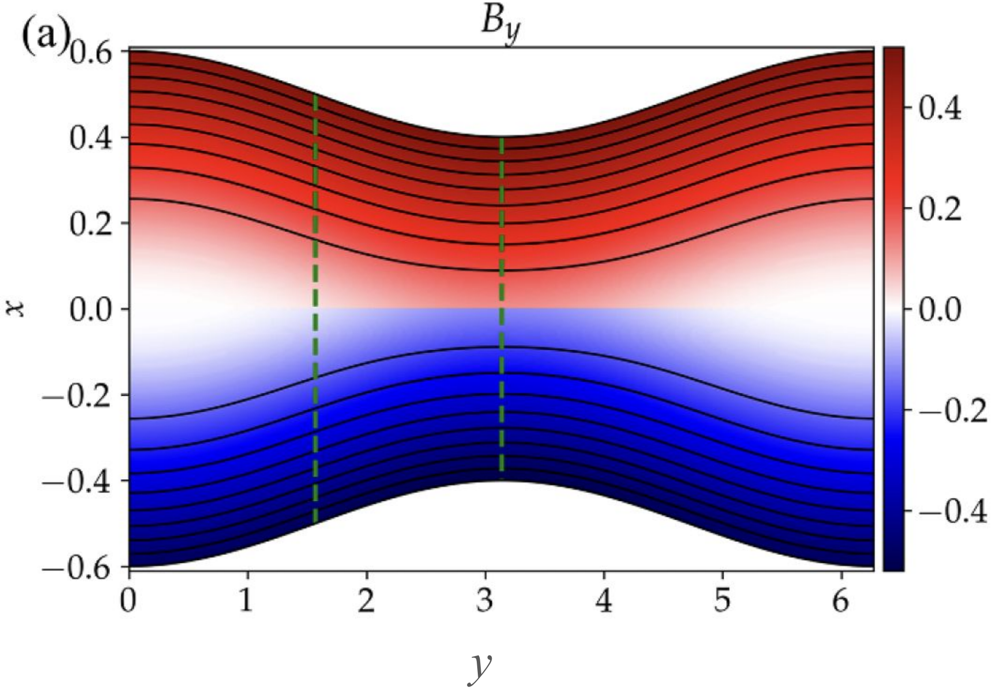
Finally, conditions are obtained additional to the magnetostatic equations appropriate to the steady state of a plasma slowly diffusing across a magnetic field out of a topologically toroidal region.

Laid the foundations for understanding MHD equilibria of toroidal plasmas (tokamaks and stellarators).

Presented a variational method for MHD equilibria. Some widely used codes today use variational formulations.

Kruskal-Kulsrud 1958 was foundational. Grad 1967 pointed out a problem: start from ideal MHD equilibria with nested flux surfaces with axisymmetric boundary, and slowly deform boundary to desired 3D shape: generically develops singular (infinite) current densities at every rational surface (and there are an infinite number…).
(ψ is continuous, but $\nabla\psi$ is discontinuous.)

Remains unsolved mystery. Can make singularities smaller (less infinite) by tailoring boundary. Most stellarator codes still assume perfectly nested smoothly-varying surfaces



Hahm-Kulsrud-Taylor 1985 paradigm problem: slowly modify 2D bdy. (Sadly, Bryan Taylor passed away Thursday)

Some progress: Per Helander 2014 review
http://dx.doi.org/10.1088/0034-4885/77/8/087001
Yi-Min Huang et al. 2023 (fig. at right)
https://doi.org/10.1088/1361-6587/acb382

CGL/Kulsrud Kinetic-MHD (gyrokinetic prehistory)

Chew-Goldberger-Low (1956, unpublished Los Alamos report): MHD fluid equations systematically closed by pressure tensor from MHD-ordered ($v_{ExB} \sim v_t$) drift-kinetic equation. (CGL published only the simplified "CGL" $p_{\|}$, $p_{\perp}$ closure approximation.)

Basic ordering: large charge limit "*e >> 1*", or:

$$\epsilon \sim \frac{\text{frequency}}{\text{gyrofrequency}} \sim \frac{\omega}{\Omega_c} \sim \frac{\text{gyroradius}}{\text{gradientLength}} \sim \frac{\rho}{L} \ll 1$$

CGL Kinetic-MHD published with clear derivation in Kulsrud (1962, 1983), based on earlier work also by Kruskal & Oberman, & by Rosenbluth & Rostoker (Cowley led a Princeton graduate journal club in early 80's that covered Kulsrud 62.) (Russell told me he was "only the messenger", but his derivation clarified and fixed important details about E||, etc.)

R. M. Kulsrud, in *Proc. of the Int. School of Physics Enrico Fermi, Course XXV, Advanced Plasma Theory*, edited by M. N. Rosenbluth (North Holland, Varenna, Italy, 1962).R. M. Kulsrud, in *Handbook of Plasma Physics*, edited by M. N. Rosenbluth and R. Z. Sagdeev (North Holland, New York, 1983). see also summary in P. B. Snyder, G. W. Hammett, W. Dorland, Phys. Plasmas **4** (1997), 3974.

Original papers and my notes on Kulsrud's various forms of Kinetic-MHD:
https://drive.google.com/drive/folders/1ruEHhhHn1FWu1j0rqdKH_EyTU8lsENx5

CGL/Kulsrud Kinetic-MHD

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{U}) = 0$$

$$\rho \left(\frac{\partial \boldsymbol{U}}{\partial t} + \boldsymbol{U} \cdot \boldsymbol{\nabla} \boldsymbol{U} \right) = \frac{(\boldsymbol{\nabla} \times \boldsymbol{B}) \times \boldsymbol{B}}{4\pi} - \boldsymbol{\nabla} \cdot \boldsymbol{P}$$

$$\frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{\nabla} \times (\boldsymbol{U} \times \boldsymbol{B})$$

$$\boldsymbol{P} = p_\perp \boldsymbol{I} + (p_{||} - p_\perp) \hat{\boldsymbol{b}} \hat{\boldsymbol{b}}$$

$$p_\perp = \sum_s \frac{m_s}{2} \int f_{0s} v_\perp^2 d^3 v$$

$$p_{||} = \sum_s m_s \int f_{0s} \left(v_{||} - \boldsymbol{U} \cdot \hat{\boldsymbol{b}} \right)^2 d^3 v$$

and a quasineutrality constraint that leads to:

$$E_{||} = \sum_s (4\pi e_s / m_s) \hat{\mathbf{b}} \cdot \nabla \cdot \mathbf{P}_s / \sum_s \omega_{ps}^2$$

$$\boldsymbol{v}_E = \boldsymbol{U}_\perp$$

$$\frac{\partial f_{0s}}{\partial t} + (v_{||} \hat{\boldsymbol{b}} + \boldsymbol{v}_E) \cdot \boldsymbol{\nabla} f_{0s} + \left(\frac{e_s}{m_s} E_{||} - \mu \hat{\boldsymbol{b}} \cdot \boldsymbol{\nabla} B - \hat{\boldsymbol{b}} \cdot \frac{D \boldsymbol{v}_E}{Dt} \right) \frac{\partial f_{0s}}{\partial v_{||}} = 0$$

Kulsrud practical fusion work w/ Fokker-Planck codes (w/ Y-C Sun et al.)

VOLUME 31, NUMBER 11 PHYSICAL REVIEW LETTERS 10 SEPTEMBER 1973

Runaway Electrons in a Plasma*

Russell M. Kulsrud, Yung-Chiun Sun, Niels K. Winsor,† and Henry A. Fallon
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 20 June 1973)

The electron runaway rate in a uniform plasma under a uniform electric field is calculated by solving the Fokker-Planck equation numerically. Comparison with other theoretical and experimental results is made.

Nature Vol. 259 February 19 1976

Neutralised colliding beam torus

R. M. Kulsrud & D. L. Jassby
Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08540

We describe a new type of deuterium–tritium fusion reactor, consisting of two dense oppositely-directed D and T plasma components confined in a torus. These components can be formed by neutral-beam injection at moderate energies. The reactor could have significant power gain with relatively lenient plasma confinement requirements.

(building on earlier work by Furth & Jassby, showing fusion gain Q~1 can be boosted with beams. (Less useful at high Q for power plant)).)

Russell's legacy of fusion-related plasma phd. students: (plus more in astro) https://plasma.princeton.edu/education/graduate-theses

1970 Dewar, Robert L., Averaged Langrangian methods and nearly periodic motions in plasmas

1974 Johnston, Russell S, Classical induced scattering of coherent waves

1978 Hassam, Adilnawaz B.S., Characteristics and lifetime of convective plasma motions in tokamaks

1978 Adler, Edward Allen (Physics) Magnetic reconnection

1985 Cowley, Steven C., Some Aspects of Anomalous Transport in Tokamaks: Stochastic Magnetic Fields, Tearing Modes and Nonlinear Ballooning Instabilities

1985 Ho, Darwin D.-M., Transport, Convective Equilibrium, and Reactor Physics in Stellarator Type Devices

Russell’s legacy of fusion-related plasma phd. students: (plus more in astro) https://plasma.princeton.edu/education/graduate-theses

1985 Goree, John A. (with M. Ono), The Backward Electrostatic Ion-Cyclotron Wave, Fast Wave Current Drive, and Far-Infrared Laser Scattering

1986 DeVore, Carl Richard, Theory and Simulation of the Evolution of the Large Scale Solar Magnetic Field

1991 Anderson, Steven W., Limits on Galactic Dynamo Theory due to Magnetic Fluctuations

1997 Chandran, Benjamin D.G., Nonlinear Turbulent Dynamos and the Origin of the Galactic Magnetic Field

1998 Uzdensky, Dmitri A., Theoretical Study of Magnetic Reconnection

2000 Felice, GianMarco, The Diffusion of Cosmic Rays through The 90 degree Pitch Angle

Russell's legacy of fusion-related plasma phd. students: (plus more in astro) https://plasma.princeton.edu/education/graduate-theses

2000 Schekochihin, Alexander, Statistical Theory of Small-scale Turbulent Astrophysical Dynamos

2010 Wang, Yansong, (with H. Ji) Nonlinear Heating of the Reconnection Layer by Strong Lower Hybrid Instabilities

Russell liked brain teasers and toy problems…

posed puzzlers / riddles at dinners.

asked me about the "power tower":

$$f(x) = x^{x^{x^{\cdot^{\cdot^{\cdot}}}}}$$

$$x^{x^{x^{x}}} \neq ((x^x)^x)^x = x^{(x^3)}$$

$$x^{x^{x^{x}}} = x^{\left(x^{\left(x^{x}\right)}\right)}$$

For what range of x does this converge? Trick to easily make plots of f(x).

Related to the Lambert W function, a recently invented (1996) special function.

Exact solution to >5 plasma problems, including sheath structure. Dubinova Plasma Physics Reports 2004, Dubinov & Dubinova JPP 2005, doi.org/10.1017/S0022377805003788